Stockton University

Mathematical Mayhem 2015

Group Round

March 21, 2015

Name: ____

Name: ____

Name: ____

High School:____

Instructions:

- This round consists of **5** problems worth **16** points each for a total of **80** points.
- Each of the 5 problems is free response.
- Write your complete solution in the space provided including all supporting work.
- No calculators are permitted.
- This round is **75 minutes** long. **Good Luck!**

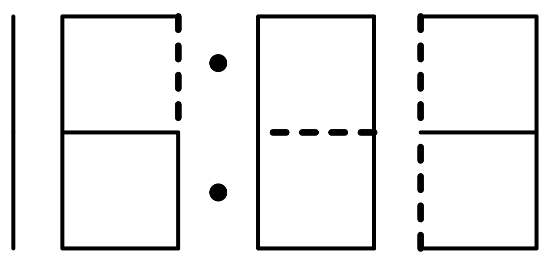
OFFICIAL USE ONLY:

Problem #	1	2	3	4	5	Total
Points Earned						

• Group Round •

Problem 1. A standard digital clock displaying the time 12:01 is shown below. This clock shows standard civilian times, and in this system the earliest time during any given day is 12:00 am and the latest time is 11:59 pm. In total 23 lights make up the clock display, 7 for each of the three digits on the right and 2 for the leftmost digit which can only display 1 or no digit. Note that if a 1 appears in any of the other 3 digits (as in the example for 12:01), the right most column of lights in that digit is used to form the 1.

Suppose that 4 of the lights that make up the clock display have burnt out, as indicated by the thick dashed segments in the figure below. That is, those lines never illuminate, even if they are needed to show the time. The non-dashed lines are fully functional, and may illuminate or not depending upon if they are needed to correctly show the time.



(a) What is the latest time that the digital clock with 4 burnt out lights shown above displays correctly?

(b) What is the earliest time that the digital clock with 4 burnt out lights shown above displays correctly?

(c) On the clock above, 4 lights have burnt out, but the clock still displays many times correctly. What is the minimum number of lights on a fully functional clock that must burn out in order for a clock to be unable to display any time correctly?

Problem 2. A group of N islands are connected by bridges. Each island has bridges to at most 3 other islands. Given any 2 islands there is a path between them that crosses at most two bridges, and bridges are allowed to go over and under other bridges. What is the largest possible value of N? Show that your answer is correct by providing a map of N islands together with the required bridges and an explanation showing why it is impossible for $N + 1$ to be connected by such bridges.

Problem 3.

(a) Consider every possible four digit number using the digits 1,2,3, and 4 where each digit is used exactly once. What is the sum of all such four digit numbers? For example, given the digits 1,2, and 3 there are 6 three digits number that use each of these digit exactly once: 123, 132, 213, 231, 312, and 321. The sum of these 6 three digit numbers is $123 + 132 + 213 + 231 + 312 + 321 = 1332$.

(b) Can you generalize this result for n digits? More specifically, for any number n from 1 to 9, write and formula that gives for the sum of every n digit number that uses each of $1, 2, \ldots, n$ exactly once.

Problem 4. An insect travels from vertex A to vertex B on the cube shown below with edges of length 1 meter. The insect can travel from A to B on one of the following three types of paths:

- crawl along the edges of the cube, at 5 meters per minute, for the entire trip from A to B,
- crawl along the7l14 Td1574 Tf 1nutei802 11.aths:

Problem 5. An integer is said to have *no repeats*